Universality in the Gravitational Stretching of Clocks, Waves and Quantum States

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Abstract

There are discernible and fundamental differences between clocks, waves and physical states in classical physics. These fundamental concepts find a common expression in the context of quantum physics in gravitational fields; matter and light waves, quantum states and oscillator clocks become quantum synonymous through the Planck-Einstein-de Broglie relations and the equivalence principle. With this insight, gravitational effects on quantum systems can be simply and accurately analyzed. Apart from providing a transparent framework for conceptual and quantitative thinking on matter waves and quantum states in a gravitational field, we address and resolve with clarity the recent controversial discussions on the important issue of the relation and the crucial difference between gravimetery using atom interferometers and the measurement of gravitational time dilation.

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The general theory of relativity preceded the formal developments of quantum mechanics. However, de Broglie's fundamental insight of relativistic physics as the starting point for dealing with matter-light symmetry and for proposing matter waves ensured that quantum dynamics of particles and light in a gravitational field was automatically compatible in structure with general relativity [1, 2]. The issue of quantum gravity – the quantum mechanics of gravity itself – is of course on a different plane and it is yet to be understood well. Gravitational quantum dynamics and physical effects are now driving applied physics with atomic clocks and atom interferometers.

Most important of general relativistic gravitational effects, touching even everyday life through GPS and positioning, is gravitational time dilation [3]; clocks have modified rates depending on the local gravitational potential, and the effect can be measured as the difference in the rates of two clocks in different gravitational potentials. Gravitational time dilation is closely linked to gravitational redshift – the change in the measured frequency of radiation as it moves through a gravitational potential difference. Though the expression derived using Newtonian physics and energy conservation, assuming the Planck relation $E = h\nu$, and the potential difference $\Delta\phi = gl$, agrees with the general relativistic expression, the correct interpretation of gravitational redshift is understood to be that the clocks that are used to measure the frequencies at two different points separated by distance l in field g run at different rates, and hence there is difference in the frequency counts. Since frequency is a number referred to a clock, only a change in the rate of the clock can change the frequency [3].

In classical physics a clock is not equivalent to a wave even though both are related to periodic changes in practice. Though all waves can serve as clocks in principle, all clocks are not waves. Also, the concept of a classical physical state, specified by various physical quantities like momentum, energy, angular momentum etc., has nothing to do *a priori* with an oscillator or wave. However in quantum mechanics these distinctions dissolve. The central starting idea of quantum physics was wave-particle duality from which emerged the concept of a wave function and so on. A stationary state of definite energy in quantum mechanics is an 'oscillator', with free time evolution factor $\exp(-iEt/\hbar)$, with a specific frequency given by the relation $\nu = E/h$. Light obeys the relation $E = h\nu$ in spite of being treated as 'particles' or photons. And a clock, like an atomic clock, is based on transitions that obey $\Delta E = h\nu$. An important unifying idea that emerges is that *once the frequency of an oscillator is specified, its progressive phase is equivalent*

to time; there is no difference between physical time and physical phase if an oscillator is used as the basis of time measurement. Since the phase of an oscillator of every kind is equivalent to time, states, photon, and matter waves can all be interpreted as 'clocks' in quantum mechanics! However, there is a price to pay for this universality – the associated wave is an abstract and unobservable entity, manifesting only through its relation to relevant probabilities. We stress this point because it is important in the rigorous and correct interpretation of what measurement of time in a gravitational field means. It is only in a space-time interpretation of quantum physics, as in the de Brolgie-Bohm theory for example, spatial ontological status can be ascribed to the quantum wave.

The analysis of quantum dynamics in weak (laboratory) gravitational fields boils down to combining the preceding observation of quantum universality with the already well known universality of the gravitational coupling. There is just one kind of coupling of a weak gravitational field to matter: $E_g = -E\phi_g/c^2$ where E is total energy of the physical system. The potential ϕ_g is simply related to the metric component g_{00}. Since there is fundamentally no distinction between general physical evolution and quantum state evolution, general relativistic effects on a generalized quantum oscillator, whether it is an atomic clock, an electromagnetic wave or an unobservable quantum state oscillator, often called a matter-wave in position representation, will follow from this interaction $E_g = -E\phi_g/c^2$ in the Hamiltonian, with E/c^2 serving as an equivalent mass. We call this universality quantum state equivalence.

Quantum state equivalence has a unifying breadth in different calculations. For example, the general relativistic deflection of light and matter in the gravitational field can be derived in a simple manner by appealing to wave-particle duality and gravitational redshift [4]. Also, the Shapiro delay can be shown to be related to this gravitational phase delay. The gravitational part of the quantum evolution is determined by the path dependent integrated phase over the path given by $\Delta_g(x) = \int E_g(x)dt/\hbar$. For atomic clocks that work with a transition frequency ν between two stationary states, $2\pi\nu\delta T = \Delta_g$ and $E = h\nu$ and the time dilation for two clocks at points x_1 and x_2 is given by

$$\delta T = T\left[\phi(x_1) - \phi(x_2)\right]/c^2 = Tgl/c^2 \tag{1}$$

Here, $l = x_1 - x_2$. This agrees with the standard general relativistic expression.

Quantum dynamics of a system that can be in a superposition of two states with different total energy exhibits interference between the two states with different evolution frequencies. The process of creating the superposition of states of different energies by giving impulses to one of the states results in a difference in the momentum associated with the two states, and there is state separation in position space while maintaining quantum coherence, leading to position-energy entanglement. For example, coherent resonant excitation by a laser with sufficient pulse duration ($\pi/2$ in terms of the inverse Rabi frequency) that creates an equal superposition $\left(|g\rangle + |e\rangle / \sqrt{2}\right)$ of ground and excited states $|g\rangle$ and $|e\rangle$ starting with the ground state, actually generates the detailed state $|g, 0\rangle + |e, \hbar k\rangle / \sqrt{2})$. The difference in momentum $\hbar k$ develops into a separation of the states in space. The quantum phase of such entangled states in a gravitational field is particularly interesting because the coupling is universal and forms the basis of gravimeters and inertial sensors employing atom interferometry [5]. For photons in a gravitational field, the relevant coupling energy is $-h\nu\phi_g/c^2$ and for material particles it is $-m\phi_g$. Since $m \approx (10^9 - 10^{11})\, h\nu/c^2$ for neutral systems convenient for matter wave interferometry, like neutrons and atoms, the sensitivity of matter wave interferometry is a whopping factor 10^{10} higher than optical interferometry in situations involving gravitational and inertial sensing.

For material particles, the gravitational energy is $E_g = -m_g\phi_g$ with the gravitational mass explicitly appearing in the quantum phase. However, as stressed in references [1, 2], there is full compatibility with the classical equivalence principle. Since the quantum phase is proportional to the product of this energy and the time spent in the potential, $t \simeq l/v$, where l is the spatial scale and v the velocity of the particle, the accumulated phase in each path is

$$\Delta_g \simeq E_g t/\hbar = -m_g\phi_g l/v\hbar \tag{2}$$

We can rewrite this expression, using the relation between the inertial mass and the de Broglie wavelength in quantum theory, $\lambda_{dB} = 2\pi\hbar/m_i v$, as

$$\Delta_g = -m_g\phi_g l m_i \lambda_{dB}/\hbar^2 \tag{3}$$

or as

$$\Delta_g = -m_g\phi_g l/m_i v^2 \lambda_{dB} = -\left(\frac{m_g}{m_i}\right)\left(\frac{E_g}{2E_{kin}}\right)\left(\frac{l}{\lambda_{dB}}\right) \tag{4}$$

The expression is particularly interesting due to the scaling expressed in terms of the kinetic energy and the wavelength, and more importantly due to

the appearance of the ratio of the gravitational and the inertial mass. We note that *in none of these expressions the mass term can be eliminated without explicitly assuming the perfect validity of the Equivalence principle.* This is crucial in the interpretation of measurements with atom interferometer. For example, to rewrite the mass term in the equation 2 as a frequency one needs to assume that the gravitational mass can be replaced with the inertial mass since all quantum frequencies refer to the inertial mass appearing in the law of dynamics. Not recognizing this subtle and important point can lead to misinterpretation of gravitational effects on quantum states.

For atom interferometry involving internal states of an atom with energies E_i, the relevant gravitational energy is $E_g = -(m_g + E_i/c^2)\phi_g$. Usually, $E_i/c^2 \ll m$, and can be ignored. For example, in the case of hyperfine transitions that define a typical primary clock, $E_i/c^2 \simeq 10^{-15}m$. In most situations, the phases arising from $E_i\phi_g/c^2$ in the two paths are equal due to the application of a π pulse that inverts the quantum states in propagation resulting in the cancellation of this contribution. Then the phase of the relevant quantum state is $\Delta_g = E_g T = -m_g\phi_g T$. It is important to note that there is no reference to any 'wave' or 'frequency' in this expression. However, one can write this in terms of phase shifts on *fictitious waves in real space*, with a projected and assumed spatiotemporal correspondence with quantum states in Hilbert space in some interpretations of quantum mechanics. The quantum state of the slow atoms is characterized by a de Broglie wave with $\lambda = h/p \simeq h/m_i v \sim 10^{-7}m$ in the case of cold atom interferometers. In contrast, the Compton wavelength is $\lambda_c = h/m_i c \sim 10^{-17}m$, which is a notional quantity and not anything physical in the context of nonrelativistic cold atoms. It is easy to demonstrate that the Compton frequency $\omega_c = m_i c^2/h$ is just a unit conversion for mass and not relevant for the interfering wave by actually forming the spatial fringe pattern; *the fringe spacing corresponds to the de Broglie wavelength $\lambda = h/p$ and not to the Compton wavelength.* The distinction becomes important and crucial if the phase shift of the fictitious wave is interpreted as the gravitational time dilation of a clock. The smallest time dilation factor than can be measured is $\delta T = \Delta_g/\omega$ where ω is the frequency of the clock oscillator. Therefore, the expression for the gravitational phase shift can be re-written to obtain a seemingly exceptional sensitivity for the measurement of gravitational time dilation if we imagine the moving atom as a real 'Compton wave clock' in space and time, in spite of the impossibility of operations like synchronization and resetting to another standard primary clock [7]. This is arguably much less rigorous than imagining it as

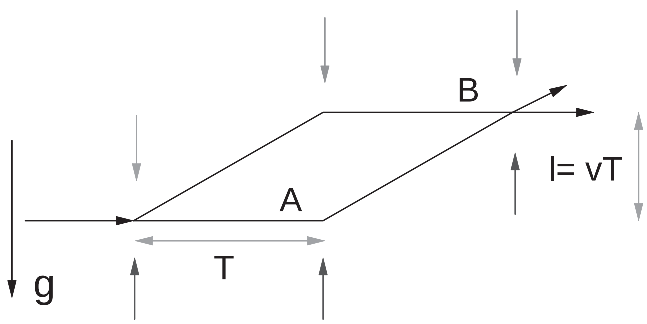

Figure 1: The space-time diagram of an atom interferometer. The laser pulses, Pi/2, Pi and Pi/2 in NMR terminology, create the superposition of hyperfine states that separate and recombine in space due the differential momentum imparted.

a real de Broglie wave with wavelength h/p propagating in real space.

We stress the important point that a physical clock should admit standard clock operations relative to a primary standards and for this it is necessary that the oscillator phase is directly accessible for comparison. For the atom interferometer gravimeter, this phase is manifested in the population of either of the hyperfine states after recombination, determined by the *phase difference* imprinted gravitationally due to the difference in the interaction energy, $-m(\phi_g(x_1) - \phi_g(x_2))$. Hence we cannot treat each of the individual wavepackets as individual clocks, just as the two-state quantum superposition separated in a Stern-Gerlach magnet is not two individual physical systems. They do not even exist in space as physical reality, except in certain non-standard interpretations of quantum mechanics.

The geometry relevant for atom interferometry is indicated in figure 1. The gravitational phase difference is simply the difference in phase accumulated over the path sections A and B with spatial separation l and temporal extent T, with gravitational potentials $\phi_g(A)$ and $\phi_g(B)$.

With a change of unit from mass to frequency, the phase shift is,

$$\Delta_g(x) = -m\phi_g(x)T/\hbar = -\frac{mc^2}{\hbar c^2}\phi_g(x)T = -\omega_c\phi_g(x)T/c^2 \tag{5}$$

Here, the coordinate x distinguishes the two paths. Gravitationally rigorous

treatment [6] gives a different expression that reveals clearly the underlying nontrivial assumption of the validity of the equivalence principle in replacing 'mass' with the Compton frequency;

$$\Delta_g(x) = -m_g\phi_g(x)T/\hbar = -\left(\frac{m_g}{m_i}\right)\frac{m_ic^2}{\hbar c^2}\phi_g(x)T = -\left(\frac{m_g}{m_i}\right)\omega_c\phi_g(x)T/c^2 \tag{6}$$

Even after assuming the equivalence principle, the appearance of the 'relativistic' Compton frequency is fictitious. To see this we note that the differential phase shift can be written as

$$\delta\Delta_g = -m\left(\phi_g(x_1) - \phi_g(x_2)\right)T/\hbar = -\frac{mv}{v}glT/\hbar = -\frac{gT^2}{\hbar/mv} = -2\pi gT^2/\lambda_{dB} \tag{7}$$

λ_{dB} is the 'relative de Broglie wavelength', calculated from the rest frame of one of the wavepackets. Since the splitting of the atomic wave packet in the interferometer is done by pulses of light with difference in momentum of $\hbar k_1 - \hbar k_2 = \hbar\kappa$ that impart a *differential recoil velocity* to the wavepackets corresponding to the two hyperfine ground states, the relation $mv = \hbar\kappa$ holds. Hence, $\kappa = 2\pi/\lambda_{dB}$. This determines the separation between the wavepackets in paths A and B as $l = vT$. In terms of the difference wave-vector κ,

$$\delta\Delta_g = -\frac{gT^2}{\hbar/mv} = -\kappa gT^2 \tag{8}$$

This is the expression for the phase shift in an atom interferometer-based gravimeter. Again, without the presumption of the equivalence principle, this expression should be written as

$$\delta\Delta_g = -\left(\frac{m_g}{m_i}\right)m_i\left(\phi_g(x_1) - \phi_g(x_2)\right)T/\hbar = -\left(\frac{m_g}{m_i}\right)\frac{gT^2}{\hbar/m_iv} = -\left(\frac{m_g}{m_i}\right)\kappa gT^2 \tag{9}$$

There are several points to note. The mass term does not disappear from the expression for the phase difference. Instead, the ratio of the gravitational to inertial mass appears along with the relative de Broglie wavelength or equivalently, the relative recoil momentum. The explicit dependence of phase is on the de Broglie wavelength and not on the Compton wavelength, as revealed in equation 7. It is clear that the gravitational phase is scaled to the de Broglie waves, as expected for slow non-relativistic atoms, and not

to the relativistic and notional Compton wave. In fact, *the phase shift is essentially the ratio of free fall distance in the gravitational field g and the de Broglie wavelength,* as dictated by the equivalence principle. In a picture projected to real space, the fringes from the interference of the de Broglie waves will 'fall' through a distance $2gT^2$ over time $2T$ and the phase shift is the ratio of this fringe shift and the 'centre of mass de Broglie wavelength' $h/(mv)/2 = 2\lambda_{dB}$.

Apart from providing a transparent analysis of the gravitational phase shift in matter-wave gravimeters, we have clarified and resolved the recent controversy in which Müller *et al.* claimed that gravimetry with the atom interferometer was equivalent to a precision measurement of gravitational redshift, the two clocks being the atomic wavepackets at A and B [7]. If true, this amounts to a gigantic improvement in the precision of clock comparison experiments and related tests of gravity theories. Although Wolf *et al.* noted that atoms could not be considered as propagating Compton waves [8], Müller *et al.* maintained their position invoking subtle details of possible small deviations from general relativity [9]. Our analysis explicitly demonstrates the dependence on the non-relativistic momentum and the de Broglie wavelength rather than on the Compton wavelength. The underlying assumption of the validity of the equivalence principle in the original claim as well as in the criticism is brought out clearly by demonstrating how the ratio of the gravitational mass to inertial mass remains in the equations for the differential quantum phase [6]. Moreover, we have argued that the observable oscillator is not the Compton wave because the 'fringes' are visible only as the oscillating population in the hyperfine states or as a spatial fringe pattern determined by the de Broglie wavelength. The Compton wave remains as a hypothesized ghost wave with no physical manifestation. Besides, individual wave packets do not qualify as clocks because the quantum states they represent have no known ontological status in space, being related to probabilities and not to actual real-time atomic positions, and they cannot be directly synchronized and adjusted relative to another primary clock.

What constitutes a clock is an important question and hard problem in the context of the universality of quantum states as oscillators that can respond to gravitational potentials in proportion to their total energy. On the one hand, all stationary quantum states have evolving phases that are modified by local gravitational potential, and on the other, only some of them can qualify as a genuine clock that can be compared to a primary clock standard. Our discussion takes a physical approach with quantitative

precision to provide clarity and a satisfactory answer.

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